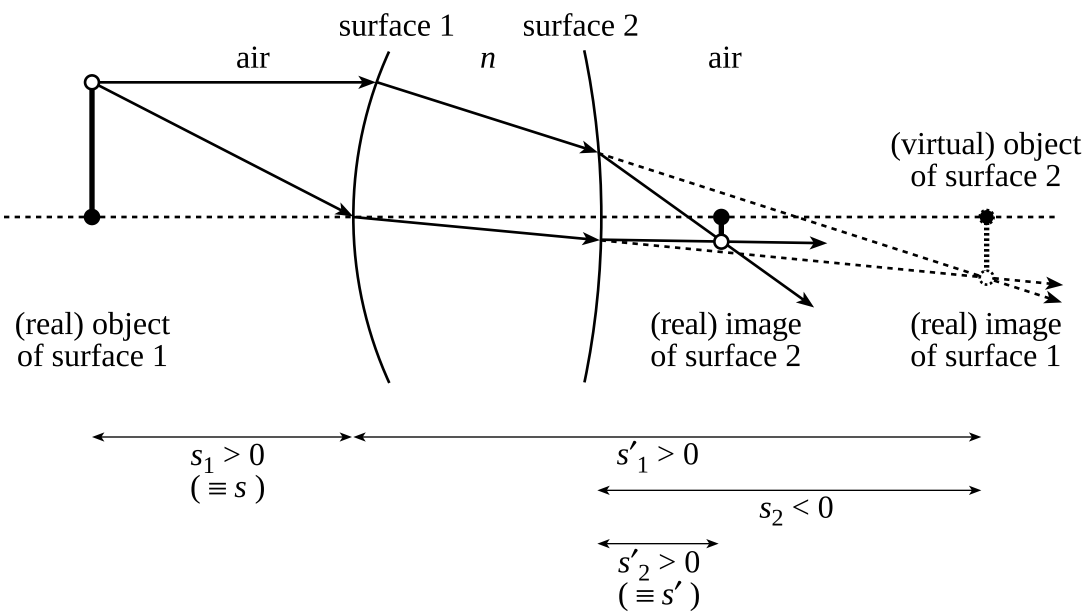
II. Light is a Ray (Geometrical Optics)

II.F. Thin Lenses

1. Image Formation by a Thin Lens

A <u>lens</u> is simply a combination of 2 refracting interfaces, at least one of which is curved. Usually the interfaces are the 2 surfaces of some piece of material that has a different index of refraction than its surroundings. Consider imaging by a standard lens in air.



We know $\dfrac{1}{s_1} + \dfrac{n}{s_1'} = \dfrac{n-1}{R_1}$ and $\dfrac{n}{s_2} + \dfrac{1}{s_2'} = \dfrac{1-n}{R_2}$ describe the object and image locations.

The lens is considered to be a <u>thin lens</u> if the piece of material is thin enough so that the light rays inside are negligibly short. That is, mathematically we can neglect the thickness of the lens so that

$$s_2 \cong -s_1',$$

where we include the minus sign because the object of surface 2 is a <u>virtual object</u>.

Adding the 2 imaging formulas above gives $\dfrac{1}{s_1} + \dfrac{1}{s_2'} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$.

If we define the first object distance $s_1 \equiv s$ and the second image distance $s'_2 \equiv s'$, then we get the imaging formula that relates the object and image distances and the radii of curvature of the 2 surfaces of the thin lens:

$$\boxed{\frac{1}{s} + \frac{1}{s'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}\ .$$

Recalling the definition of <u>Focal Length</u> f as the image distance for an object infinitely far away, or the object distance that produces an image infinitely far away, we obtain an expression for the focal length often called the Lensmaker's Equation:

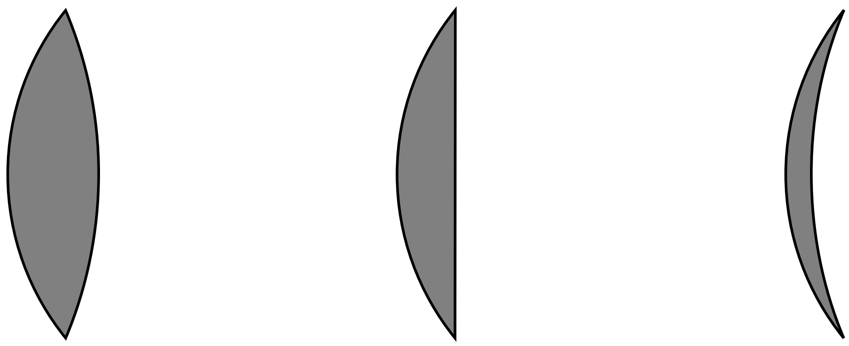
<u>Lensmaker's Equation for the Focal Length of a Thin Lens</u>: $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right).$

Combining this expression with the imaging formula we obtained above, we arrive at the

<u>Imaging Formula for a Thin Lens</u>: $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$.

2. Positive Thin Lenses

A <u>positive thin lens</u> has a *positive focal length*, $f > 0$. Positive lenses are *thicker in the middle* than they are at the edges. There are 3 standard types of positive thin lenses:



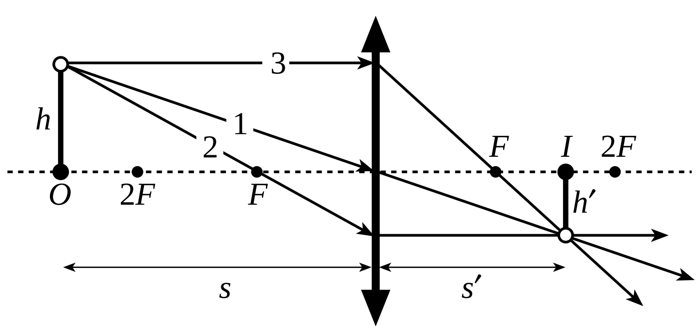
<u>biconvex</u> <u>planoconvex</u> <u>positive meniscus</u>

Depending upon where the object is located near the lens, there are 4 general relationships between the object and the image. To examine these, we apply the General Rule for imaging to the specific case of a thin lens.

<u>Rule for Locating an Image Formed by a Thin Lens</u>:

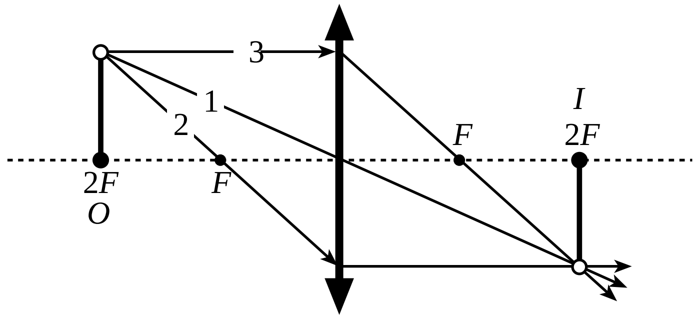
1. *Trace a ray from the tip of the object through the point of intersection of the lens and the optical axis — this ray continues undeviated.*
2. *Trace a ray from the tip of the object through the front focal point for a positive lens (back focal point for a negative lens) — this ray is bent such that it travels parallel to the optical axis.*
3. *Trace a ray from the tip of the object parallel to the optical axis — this ray is bent such that it passes through the back focal point for a positive lens (front focal point for a negative lens).*

a. <u>Object Distance Greater than 2 Focal Lengths</u> ($s > 2f$):



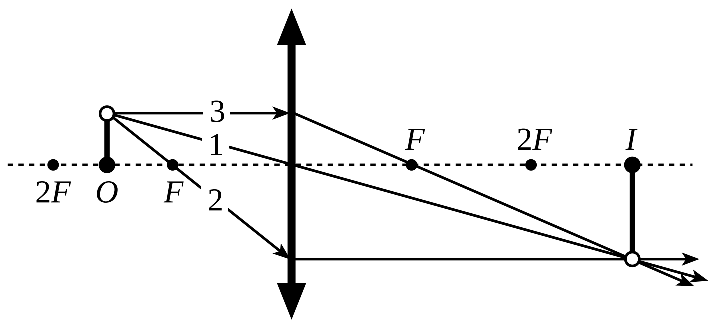
real image
magnification < 1

b. <u>Object Distance Equals 2 Focal Lengths</u> ($s = 2f$): This is “one-to-one” or “2F-to-2F” imaging.



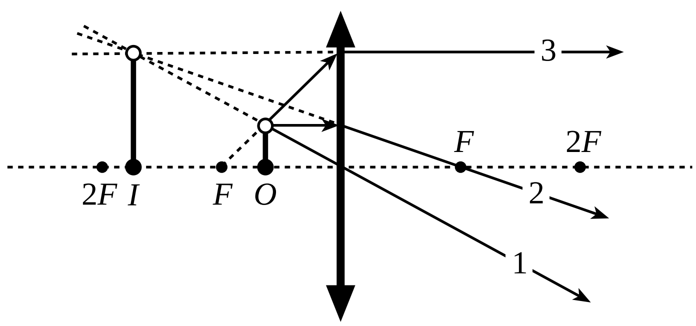
real image
magnification = 1

c. <u>Object Distance Between 1 and 2 Focal Lengths</u> ($f < s < 2f$):



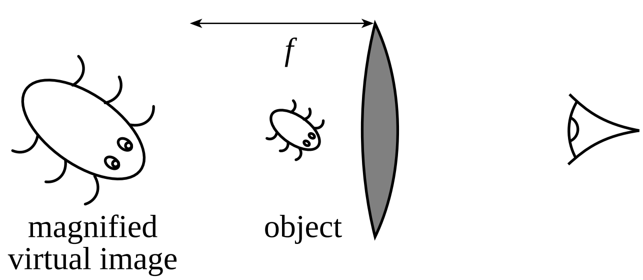
real image
magnification > 1

d. <u>Object Distance Less than 1 Focal Length</u> ($s < f$):



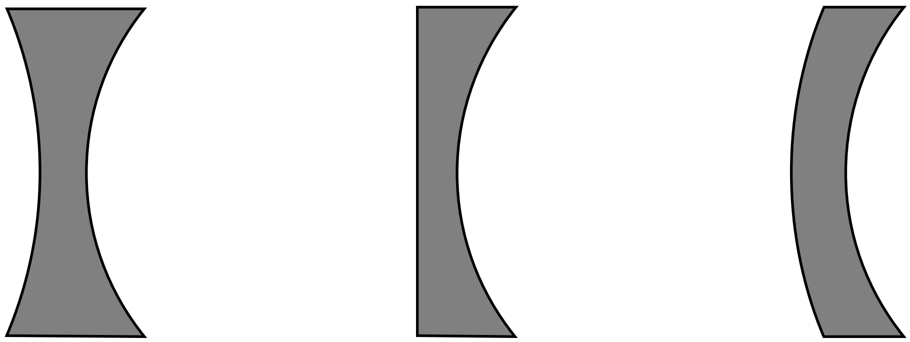
virtual image
magnification > 1

This case demonstrates how a simple magnifying glass works!



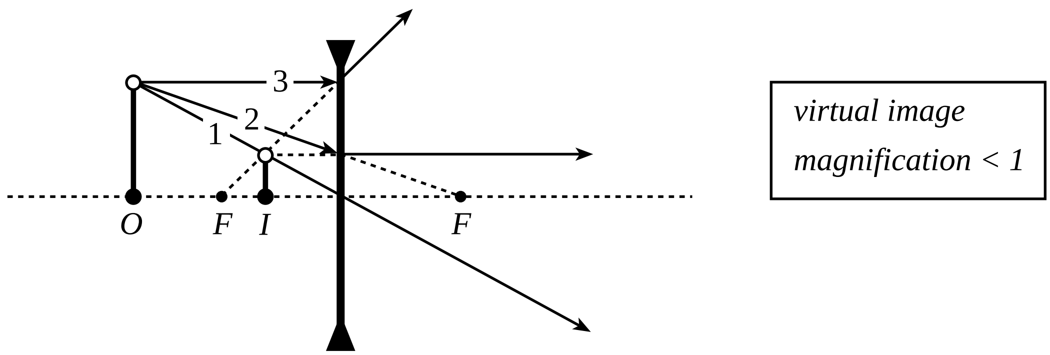
3. Negative Thin Lenses

A <u>negative thin lens</u> has a *negative focal length*, $f < 0$. Negative lenses are *thinner in the middle* than they are at the edges. There are 3 standard types of negative thin lenses:



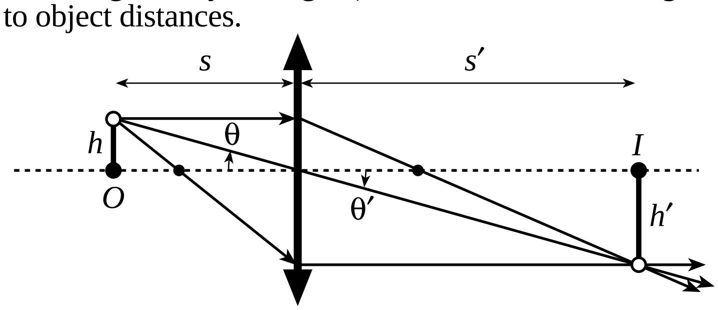
biconcave planoconcave negative meniscus

We can use the same rule described in the previous section for locating an image, except we use the opposite focal points for the second and third rays when analyzing a negative lens, as indicated.



4. Thin Lens Magnification

Just as we did for the spherical mirror, we can write the magnification associated with a thin lens in terms of both the ratio of image to object height (i.e., the definition of magnification), or in terms of the ratio of image to object distances.



Looking at the diagram, and minding the Sign Convention, we see that

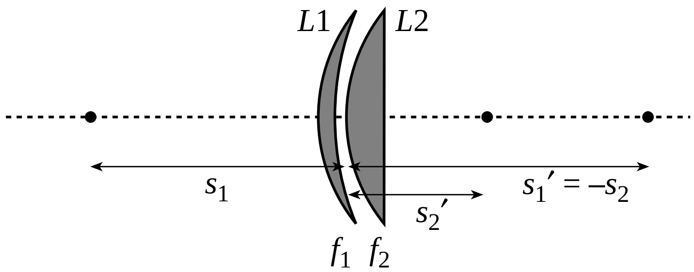
$\dfrac{h}{s} = \tan\theta = \tan\theta' = \dfrac{-h'}{s'}$ (note that $-h'$ is a positive number). Thus the magnification is

$$\boxed{m = \frac{h'}{h} = -\frac{s'}{s}}\,.$$

This result for thin lens magnification applies to both positive and negative thin lenses.

5. Thin Lenses in Contact — Powers (in Diopters) Add

What happens when we combine 2 thin lenses by placing them right next to each other?



$$L1: \quad \frac{1}{s_1} + \frac{1}{s_1'} = \frac{1}{f_1}$$

$$L2: \quad \frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f_2}$$

but by construction $s_2 = -s_1'$, so we can rewrite the $L2$ equation as $-\frac{1}{s_1'} + \frac{1}{s_2'} = \frac{1}{f_2}$.

Adding the equations for $L1$ and $L2$, we obtain $\frac{1}{s_1'} + \frac{1}{s_2'} = \frac{1}{f_1} + \frac{1}{f_2}$.

But this looks just like the equation for a single lens with a focal length f given by

$$\frac{1}{f} \equiv \frac{1}{f_1} + \frac{1}{f_2}.$$

Thus it makes sense to define a quantity called the power of a lens as:

Power of a Lens, P:	$P\,(\textit{in diopters}) = \frac{1}{f\,(\textit{in meters})}$.

So *the total power for multiple thin lenses in contact is simply the sum of the powers of each of the individual lenses:*

$$P_{total} = P_1 + P_2 + P_3 + \cdots \; .$$

By adding together "test lenses" of known powers, an optometrist can easily determine the total power (in diopters) your glasses or contact lenses should have!